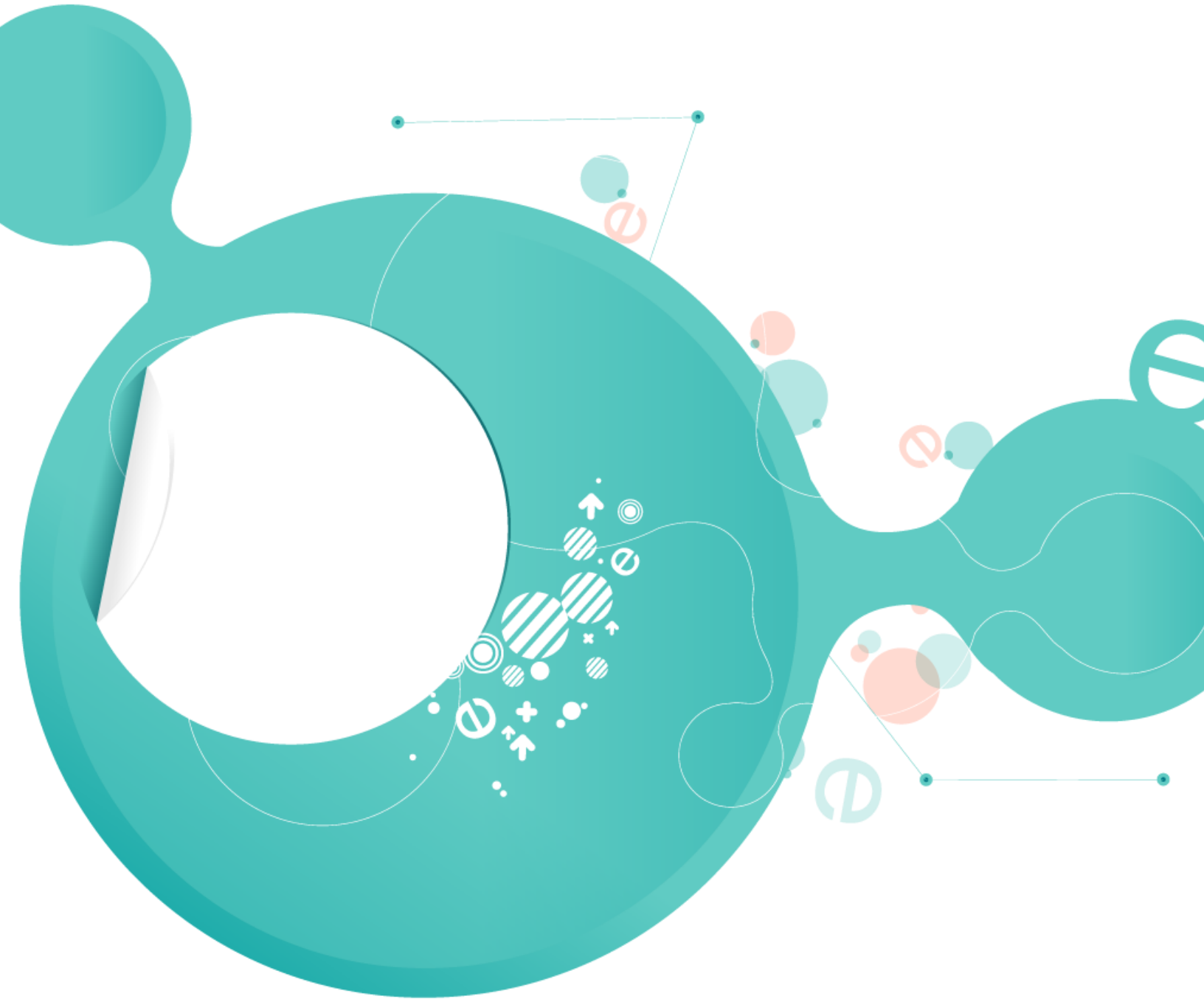
Mathematics for Economists

Tutorial Questions - Integration



Tutorial 4: Integration

ECO4112F 2011

1. $\int (x+1)^{20}\,dx$
2. $\int x\sqrt{x^2+5}dx$
3. $\int 4x^2\left(x^4+1\right)^2 dx$
4. $\int 2xe^{x^2}dx$
5. $\int \left(x^2+1\right)e^{x^3+3x}dx$
6. $\int \frac{7}{x}dx$
7. $\int \frac{\left(2x^3+3x\right)}{x^4+3x^2+7}dx$
8. $\int \sqrt{2x-1}dx$
9. $\int e^{-x/4}dx$
10. $\int \left(x^2+1\right)^2 dx$
11. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$
12. $\int \left[\frac{1}{x-1}+\frac{1}{(x-1)^2}\right]dx$
13. $\int \sin(5x-3)\,dx$
14. $\int \frac{7}{3-2x}dx$
15. $\int 2^{3-x}dx$
16. $\int \frac{9x^2+5}{3x}dx$
17. $\int \frac{3^{\ln x}}{x}dx$
18. $\int \frac{x+3}{x+6}dx$
19. $\int_{-1}^{3}\left(3x^2-x+6\right)dx$
20. $\int_{0}^{1} e^{3t}dt$
21. $\int_{0}^{1} 2x^2\left(x^3-1\right)^3 dx$
22. $\int_{-1}^{1} \sqrt[3]{x^5}dx$
23. $\int \sqrt{10^{3x}}dx$
24. $\int \frac{\ln x}{\sqrt{x}}dx$
25. $\int_{1}^{2} x\ln x dx$
26. $\int xe^{x^2}dx$
27. $\int x^2e^{2x+1}dx$
28. $\int xe^{-x}dx$
29. $\int y^3\ln y dy$
30. $\int x\sqrt{x+1}dx$
31. $\int_{1}^{2} xe^{2x}dx$
32. $\int (2x-1)\ln(x-1)\,dx$
33. $\int e^x\sin x dx$
34. $\int_{1}^{e} \sqrt{x}\ln\left(x^2\right)dx$

Tutorial 4: Integration

SELECTED SOLUTIONS

ECO4112F 2011

1. $\int (x+1)^{20}\,dx$

   Easy

2. $\int x\sqrt{x^2+5}dx$

   $= \dfrac{\left(x^2+5\right)^{3/2}}{3} + C$

3. $\int 4x^2\left(x^4+1\right)^2 dx$

   $= 4\left(\dfrac{x^{11}}{11} + \dfrac{2x^7}{7} + \dfrac{x^3}{3}\right) + C$

4. $\int 2xe^{x^2}dx$

   $= e^{x^2} + C$

5. $\int \left(x^2+1\right)e^{x^3+3x}dx$

   $= \dfrac{1}{3}e^{x^3+3x} + C$

6. $\int \dfrac{7}{x}dx$

   Easy

7. $\int \dfrac{\left(2x^3+3x\right)}{x^4+3x^2+7}dx$

   Easy

8. $\int \sqrt{2x-1}dx$

   $= \dfrac{1}{3}\left(2x-1\right)^{3/2} + C$

9. $\int e^{-x/4}dx$

   ...

10. $\int \left(x^2+1\right)^2 dx$

    $= \dfrac{x^5}{5} + \dfrac{2x^3}{3} + x + C$

11. $\int \dfrac{e^{\sqrt{x}}}{\sqrt{x}}dx$

    $= 2e^{\sqrt{x}} + C$

12. $\int \left[\dfrac{1}{x-1} + \dfrac{1}{(x-1)^2}\right] dx$

    ...

13. $\int \sin\left(5x-3\right) dx$

    $= -\dfrac{\cos\left(5x-3\right)}{5}$

14. $\int \dfrac{7}{3-2x}dx$

    Simple, but be careful of signs

15. $\int 2^{3-x}dx$

    $= -\dfrac{1}{\ln 2}2^{3-x} + C$

16. $\int \dfrac{9x^2+5}{3x}dx$

    Hint: divide through

17. $\int \dfrac{3^{\ln x}}{x}dx$

    $= \dfrac{3^{\ln x}}{\ln 3} + C$

18. $\int \dfrac{x+3}{x+6}dx$

    Hint: rewrite the top and break up into two fractions ...

19. $\int_{-1}^{3} \left(3x^2 - x + 6\right) dx$

    $= 48$

20. $\int_0^1 e^{3t}dt$

$= \frac{1}{3}\left(e^3 - 1\right)$

21. $\int_0^1 2x^2\left(x^3-1\right)^3 dx$

$= -\frac{1}{6}$

22. $\int_{-1}^1 \sqrt[3]{x^5}dx$

$= 0$

23. $\int \sqrt{10^{3x}}dx$

$= \frac{2\sqrt{10^{3x}}}{3\ln 10} + C$

24. $\int \frac{\ln x}{\sqrt{x}}dx$

$= 2\sqrt{x}\left(\ln x - 2\right) + C$

25. $\int_1^2 x\ln x dx$

$= 2\ln 2 - \frac{3}{4}$

26. $\int xe^{x^2}dx$

$= \frac{1}{2}e^{x^2} + C$

27. $\int x^2 e^{2x+1}dx$

$= \frac{e^{2x+1}}{2}\left(x^2 - x + \frac{1}{2}\right) + C$

28. $\int xe^{-x}dx$

Easy, use by parts

29. $\int y^3 \ln y dy$

$= \frac{y^4}{4}\left(\ln y - \frac{1}{4}\right) + C$

30. $\int x\sqrt{x+1}dx$

$= \frac{2}{15}\left(x+1\right)^{3/2}\left(3x-2\right) + C$

31. $\int_1^2 xe^{2x}dx$

$= \frac{1}{4}e^2\left(3e^2 - 1\right)$

32. $\int \left(2x-1\right)\ln\left(x-1\right)dx$

$= x\left(x-1\right)\ln\left(x-1\right) - \frac{x^2}{2} + C$

33. $\int e^x \sin x dx$

Don't use by parts more than twice

34. $\int_1^e \sqrt{x}\ln\left(x^2\right)dx$

Use by parts.

ECO4112F, Tutorial 4

12

$$\begin{aligned}\int\left[\frac{1}{x-1}+\frac{1}{(1-x)^2}\right]dx &= \int\frac{1}{x-1}dx+\int\frac{1}{(x-1)^2}dx \\ &= \ln(x-1)+-(x-1)^{-1}+C\end{aligned}$$

24

$$\begin{aligned}\int\frac{\ln x}{\sqrt{x}}dx &= \int(\ln x)\left(x^{-\frac{1}{2}}\right)dx \\ &= 2x^{\frac{1}{2}}\ln x+A-\int 2x^{\frac{1}{2}}\frac{1}{x}dx \\ &= 2x^{\frac{1}{2}}\ln x+A-\int 2x^{-\frac{1}{2}}dx \\ &= 2x^{\frac{1}{2}}\ln x+A-4x^{\frac{1}{2}}-B \\ &= 2\sqrt{x}(\ln x-2)+C\end{aligned}$$

25

$$\begin{aligned}\int_1^2 x\ln x dx &= \frac{1}{2}x^2\ln x\Big|_1^2-\int_1^2\frac{1}{2}x^2\frac{1}{x}dx \\ &= \left(2\ln 2-\frac{1}{2}\ln 1\right)-\frac{1}{4}x^2\Big|_1^2 \\ &= 2\ln 2-\frac{3}{4}\end{aligned}$$

34

$$\begin{aligned}\int_1^e\sqrt{x}\ln x^2dx &= \int_1^e 2x^{\frac{1}{2}}\ln x dx \\ &= \frac{4}{3}x^{\frac{3}{2}}\ln x\Big|_1^e-\int_1^e\frac{4}{3}x^{\frac{3}{2}}\frac{1}{x}dx \\ &= \frac{4}{3}e^{\frac{3}{2}}-\frac{8}{9}x^{\frac{3}{2}}\Big|_1^e \\ &= \frac{4}{3}e^{\frac{3}{2}}-\frac{8}{9}e^{\frac{3}{2}}+\frac{8}{9} \\ &= \frac{4}{9}e^{\frac{3}{2}}+\frac{8}{9}\end{aligned}$$