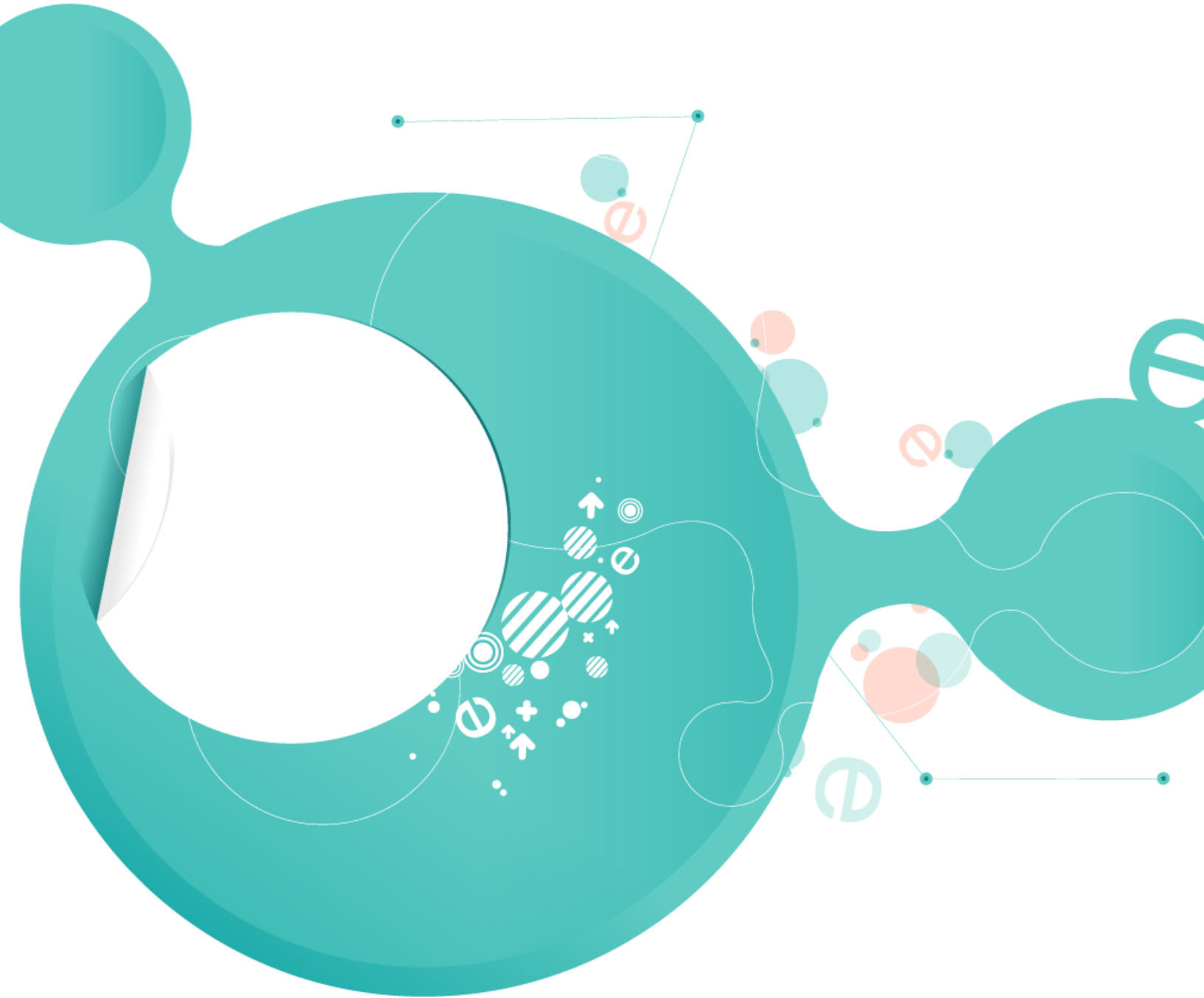




# Mathematics for Economists

## Tutorial Questions - Basic Concepts



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# Maths for Economists

## Tutorial 0: Revision of Basic Concepts

ECO4112F 2011

In the following paired statements, let  $p$  be the first statement and  $q$  the second. Indicate for each case whether  $q$  is necessary or sufficient or both necessary and sufficient for  $p$  (i.e. whether  $p \Rightarrow q$ ,  $p \Leftarrow q$ ,  $p \Leftrightarrow q$ ) or whether none of these apply.

1. Alice has a brother-in-law. Alice is not an only child.
2. Alice's daughter is married. Alice is a parent.
3. Water contracts when it freezes. Not all substances expand when they freeze.
4. Some women pay taxes. Some taxpayers are women.
5. Everybody loves somebody (or other). There is some (one) person who is loved by everyone.

Simplify the following expressions:

- |                                |  |  |
|--------------------------------|--|--|
| 6. $x^5 \times x^4$            | 15. $\frac{(x^{1/2} \times x^{1/3})}{x^{2/3}}$ | 22. $\left(-\frac{8}{27}\right)^{4/3}$ |
| 7. $\frac{p^3}{p^5}$           | 16. $\frac{y^{11}}{y^2}$                       | 23. $(64a^3)^{2/3}$                    |
| 8. $x^a \times x^b \times x^c$ | 17. $x^6 x^8$                                  | 24. $\frac{x^{-2} y^3}{z^{-2}}$        |
| 9. $s^2 \times r^2 \times t^2$ | 18. $x^{11} x^{-5}$                            | 25. $(x^5 y^8)^5$                      |
| 10. $(x^{1/2})^2$              | 19. $x^{2/5} x^{3/5}$                          | 26. $(x^{5/9} y^{4/3})^{18}$           |
| 11. $x^4 + y^4$                | 20. $xx^{1/2}$                                 | 27. $x^{-1} + y^{-1}$                  |
| 12. $s^5 - s^3$                | 21. $\left(\frac{1}{4}\right)^{3/2}$           | 28. $7x^{-2} + (7x)^{-2}$              |
| 13. $x^7 + x^2$                |  |  |
| 14. $\frac{s^5}{s^3}$          |  |  |

Simplify the following expressions:

29.  $\log_{10} 1000$

30.  $\log_3 81$

31.  $\log_2 16$

32.  $\ln e^4$

33.  $\ln e$

34.  $\ln e^4 + \ln e^7$

35.  $\ln \frac{1}{e^3}$

36.  $\log_e \frac{1}{e^2}$

37.  $\log_{10} (100)^{14}$

38.  $\ln ABe^{-4}$

39.  $\ln \frac{3}{B}$

40.  $\ln x - \ln(x + 3)$

41.  $\ln 3 + \ln 7 - \ln 2 - 2 \ln 4$

42.  $\ln e^{3x}$

43.  $\log_{10} 1 + \log_{10} 1000$

44.  $\ln(x(x + 1)^2)$

45.  $\ln \left( \frac{x}{x + 1} \right)^3$

46.  $\ln \frac{x}{(x + 1)(x + 2)}$

47.  $\ln \left( \frac{1}{x + 2} \sqrt[5]{\frac{x^2}{x + 1}} \right)$

48.  $9 \log_{10} 7 + 5 \log_{10} 23$

49.  $e^{4 \ln 3 - 3 \ln 4}$

Solve for  $x$  :

50.  $10^{\log_{10} x^2} = 25$

52.  $10^{\log_{10} x^2} = 4$

51.  $e^{\ln(2x)} = 5$

53.  $e^{3 \ln x} = 8$

54. If  $z = x^{1/2}y^{1/4}$  and  $y = 2x$ , find  $x$  and  $y$  in terms of  $z$ .

55. A firm's output  $Y$  is related to capital input  $K$  and labour input  $L$  by the production function

$$Y = 2K^{2/3}L^{1/3}$$

Suppose initially  $K = a$  and  $L = b$ . Find the percentage increase in  $Y$  resulting from 1% increases in both  $K$  and  $L$ . What happens when the changes are each 100%? Can you formulate a general result?

56. Use the properties of logarithms to show that  $\log_a x = 1/\log_x a$ .

57. A firm's output  $Y$  is related to capital input  $K$ , labour input  $L$  and natural resource input by the production function

$$Y = 2K^{1/2}L^{1/3}R^{1/6}$$

Write down a linear relationship between the logarithms of  $Y, K, L, R$ .

Find the rational roots, if any, of the following:

58.  $x^2 - 8x + 15 = 0$

59.  $x^3 - 4x^2 + x + 6 = 0$

60.  $x^3 + \frac{3}{4}x^2 - \frac{3}{8}x - \frac{1}{8} = 0$

Solve for equilibrium values:

61.  $Q_d = Q_s$

$$Q_d = 24 - 2P$$

$$Q_s = -5 + 7P$$

62.  $Q_d = Q_s$

$$Q_d = 51 - 3P$$

$$Q_s = 6P - 10$$

63.  $Q_d = Q_s$

$$Q_d = 3 - P^2$$

$$Q_s = 6P - 4$$

64. Consider the following national income model:

$$Y = C + I_0 + G_0$$

$$C = a + bY \quad a > 0, 0 < b < 1$$

where  $Y$  and  $C$  represent the endogenous variables national income and consumption expenditure respectively, and  $I_0$  and  $G_0$  stand for exogenously determined investment and government expenditure respectively.

- (a) What do the parameters  $a$  and  $b$  in the consumption function represent?
- (b) Solve for the equilibrium values of income ( $Y^*$ ) and consumption ( $C^*$ ) using the substitution method. Be sure to specify any restrictions that are necessary for your solution values to hold.

65. Sketch the graph of the following supply function for petrol:

$$q = \frac{1}{4}p^4$$

where  $p$  represents price and  $q$  represents quantity.

Now consider the function

$$q = 8p^{-1}$$

where  $p$  is positive. Explain why  $q$  decreases as  $p$  increases. If this function is the demand function for petrol, find the equilibrium price and quantity.

Also find the equilibrium price and quantity by expressing the supply and demand functions in log-linear form and solving the resulting linear equations simultaneously.

# Maths for Economists

## Tutorial 0: Revision of Basic Concepts

### SELECTED SOLUTIONS

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1. For Alice to have a brother-in-law it is not necessary that Alice is not an only child so  $p \not\Rightarrow q$ .

If Alice is not an only child, then it is not sufficient to conclude that Alice to has a brother-in-law, so  $p \not\Leftarrow q$ .

Thus none of these apply.

2. For Alice's daughter to be married, it is necessary that Alice is a parent, so  $p \Rightarrow q$ .

If Alice is a parent, it is not sufficient to conclude that Alice's daughter is married, so  $p \not\Leftarrow q$ .

Thus  $q$  is necessary but not sufficient for  $p$ , i.e.  $p \Rightarrow q$ .

3. If water contracts when it freezes, then it is necessary that not all substances expand when they freeze, so  $p \Rightarrow q$

If not all substances expand when they freeze, then it is not sufficient to conclude that water contracts when it freezes, so  $p \not\Leftarrow q$ .

Thus  $q$  is necessary but not sufficient for  $p$ , i.e.  $p \Rightarrow q$ .

4. If some women pay taxes, then it is necessary that some taxpayers are women, so  $p \Rightarrow q$ .

If ome taxpayers are women, then it is sufficient to conclude that some women pay taxes, so  $p \Leftarrow q$ .

Thus  $q$  is necessary and sufficient for  $p$ , i.e.  $p \Leftrightarrow q$ .

5. For everybody to love somebody (or other), it is not necessary that there is some (one) person who is loved by everyone, so  $p \not\Rightarrow q$ .

If there is some (one) person who is loved by everyone, then it is sufficient to conclude that everybody loves somebody (or other), so  $p \Leftarrow q$ .

Thus  $q$  is sufficient but not necessary for  $p$ , i.e.  $p \Leftarrow q$ .

- |                          |   |  |
|--------------------------|---|--|
| 6. $x^9$                 | 14. $s^2$                                 | 22. $\frac{16}{81}$                                      |
| 7. $\frac{1}{p^2}$       | 15. $x^{1/6}$                             | 23. $16a^2$  |
| 8. $x^{a+b+c}$           | 16. $y^9$                                 | 24. $\frac{y^3z^2}{x^2}$                                 |
| 9. $(srt)^2$             | 17. $x^{14}$                              | 25. $x^{25}y^{40}$                                       |
| 10. $x$                  | 18. $x^6$                                 | 26. $x^{10}y^{24}$                                       |
| 11. Cannot be simplified | 19. $x$                                   | 27. $\frac{x+y}{xy}$                                     |
| 12. Cannot be simplified | 20. $x^{3/2}$                             | 28. $\frac{7}{x^2} + \frac{1}{49x^2}$                    |
| 13. Cannot be simplified | 21. $\frac{1}{8}$                         |  |
| 29. 3                    | 37. 28                                    | 45. $3(\ln x - \ln(x+1))$                                |
| 30. 4                    | 38. $\ln A + \ln B - 4$ note <sup>1</sup> | 46. $\ln x - \ln(x+1) - \ln(x+2)$                        |
| 31. 4                    | 39. $\ln 3 - \ln B$                       | 47. $-\ln(x+2) + \frac{2}{5}\ln x - \frac{1}{5}\ln(x+1)$ |
| 32. 4                    | 40. $\ln \frac{x}{x+3}$                   | 48. $\log_{10}(7^9(23^5))$                               |
| 33. 1                    | 41. $\ln \frac{21}{32}$                   | 49. $\frac{81}{64}$                                      |
| 34. 11                   | 42. $3x$                                  |  |
| 35. -3                   | 43. 3                                     |  |
| 36. -2                   | 44. $\ln x + 2\ln(x+1)$                   |  |
| 50. $x = \pm 5$          | 51. $x = \frac{5}{2}$                     | 52. $x = \pm 2$  |
|                          |   | 53. $x = 2$  |

54.  $x = \frac{z^{4/3}}{2^{1/3}}, y = 2^{2/3}z^{4/3}$

55. If  $K$  and  $L$  both increase by  $s\%$ ,  $Y$  increases by  $s\%$ .

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<sup>1</sup>The question was ambiguous and should have read  $\ln(ABe^{-4})$ .

56. ...

$$57. \ln Y = \ln 2 + \frac{1}{2} \ln K + \frac{1}{3} \ln L + \frac{1}{6} \ln R$$

58. 3, 5

59. -1, 2, 3

60.  $-1, \frac{1}{2}, -\frac{1}{4}$

$$61. P^* = 3\frac{2}{9},$$

$$Q^* = Q_d^* = Q_s^* = 17\frac{5}{9}$$

$$62. P^* = \frac{61}{9}, Q^* = \frac{92}{3}$$

63.  $P^* = 1$  ( $P^* = -7$  not valid),

$$Q^* = 2$$

64. (a)  $a$  is autonomous consumption,  $b$  is marginal propensity to consume

$$(b) Y^* = \frac{a + I_0 + G_0}{1 - b}, \quad a > 0, 0 < b < 1$$

$$C^* = \frac{a + b(I_0 + G_0)}{1 - b}, \quad a > 0, 0 < b < 1$$

65. ...