2013

ECO 4112 F

Katherine Eyal

Lecture 3 $\boxed{7.5, 7.6}$

Ch 7.5 | 7.6 | 8
P 170 – 218

Applications to Comparative Static (CS) analysis

Market Model

$Q = a - bP$ $a, b > 0$ Demand
$Q = -c + dP$ $c, d > 0$ Supply

Solns

$P^* = \dfrac{a+c}{b+d}$ $Q^* = \dfrac{ad - bc}{b+d}$ (Check)

= Reduced form
= 2 endog vars = fn (parameters)

What is $\dfrac{\partial P^*}{\partial a}$? How is $\dfrac{\partial P}{\partial a}$ different

how does EQM quantity change w.r.t Δa

How does P in DD fn (or SS fn) change w.r.t a, without reference to SS (or DD)

$\frac{\partial P^*}{\partial \_}$ & $\frac{\partial Q^*}{\partial \_}$ ← comparative static derivatives

Look at $\frac{\partial P^*}{\partial \_}$

$\frac{\partial P^*}{\partial a} = \frac{1}{b+d}$ $\qquad$ $\frac{\partial P^*}{\partial b} = -\frac{(a+c)}{(b+d)^2}$

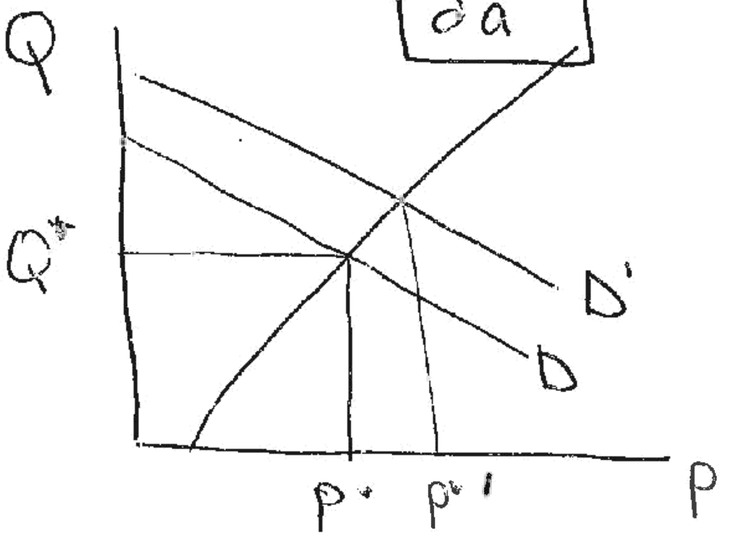
$\frac{\partial P^*}{\partial c} = \frac{1}{b+d} \quad \left(= \frac{\partial P^*}{\partial a} \right)$

$\frac{\partial P^*}{\partial d} = -\frac{(a+c)}{(b+d)^2} \quad \left(= \frac{\partial P^*}{\partial b} \right)$

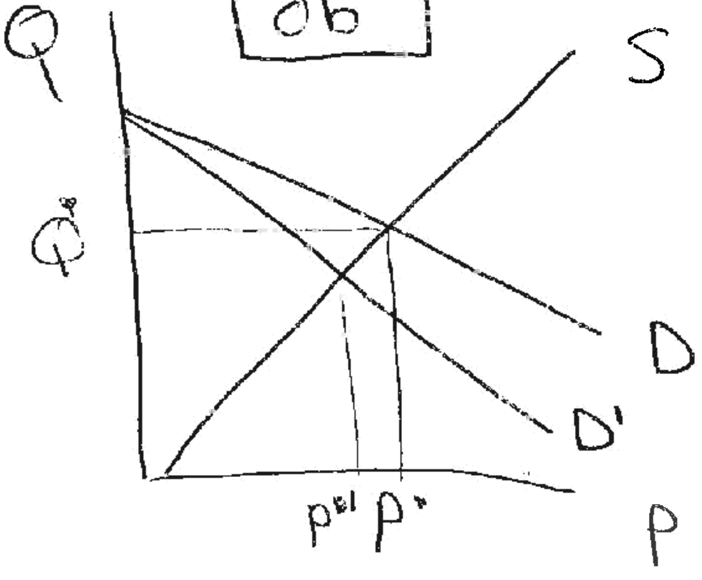
$\frac{\partial P^*}{\partial a} = \frac{\partial P^*}{\partial a} > 0$ $\qquad$ $\frac{\partial P^*}{\partial b} = \frac{\partial P^*}{\partial d} < 0$

P171

$\boxed{\frac{\partial P^*}{\partial a}}$



$\boxed{\frac{\partial P^*}{\partial b}}$



3

Check (P172) You can create the other 2 graphs.

- You must know which curve changes, & why it swivels or shifts.
- Why use differentiation (diff$^{\underline{n}}$) if graphs work?
  - L no restriction on dimension
  - L results generalisable

National Income Model P172

$$Y = C + I_0 + G_0 \qquad \alpha > 0, 0 < B < 1$$
$$C = \alpha + B(Y - T) \qquad \gamma > 0 ; 0 < \delta < 1$$
$$T = \gamma + \delta Y$$

- Why are the signs of the parameters as assumed?

B = mpc, α = autonomous cons$^{\underline{n}}$

γ = tax revenue not from Y,

δ = tax rate

exog variables - $I_0, G_0 \geq 0$.

$$Y^* = \frac{\alpha - B\gamma + I_0 + G_0}{1 - B + B\delta}$$ 4

(check)

We can find $\dfrac{\partial Y^*}{\partial G_0, \gamma, \delta, \alpha, B, I}$

$$\frac{\partial Y^*}{\partial G_0} = \frac{1}{1 - B + B\delta} > 0$$

= govt expenditure multiplier

$$\frac{\partial Y^*}{\partial \gamma} = \frac{-B}{1 - B + B\delta} < 0$$

= non income tax multiplier

$$\frac{\partial Y^*}{\partial \delta} = \frac{-B(\alpha - B\gamma + I_0 + G_0)}{(1 - B + B\delta)^2}$$

$$= \frac{-BY^*}{(1 - B + B\delta)} < 0.$$

if raise tax rate δ, $Y^* \downarrow$

$$\frac{\partial Y}{\partial G_0} = 1 \quad , \quad \frac{\partial Y^*}{\partial G_0} = \frac{1}{1 - B + B\delta}$$

Why different?

5

Input Output Model.

Solution to an open I-O model is $x^* = (I - A)^{-1} d$

Let $V = [v_{ij}] = (I - A)^{-1}$

$\therefore$ 3 industry economy:

$x^* = Vd$

$$\begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} = \underset{3\times 3}{\begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}$$

$$\frac{\partial x_j^*}{\partial d_k} = V_{jk} \quad (j, k = 1, 2, 3)$$

Why?

$$\begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} = \underset{3\times 1}{\begin{bmatrix} V_{11} d_1 + V_{12} d_2 + V_{13} d_3 \\ V_{21} d_1 + V_{22} d_2 + V_{23} d_3 \\ V_{31} d_1 + V_{32} d_2 + V_{33} d_3 \end{bmatrix}}$$

$\therefore \dfrac{\partial x_1^*}{\partial d_1} = V_{11}$ $\quad \dfrac{\partial x_1^*}{\partial d_2} = V_{12}$ $\quad \dfrac{\partial x_2^*}{\partial d_2} = V_{22}$

$\dfrac{\partial x_2^*}{\partial d_1} = V_{21}$ $\quad \dfrac{\partial x_3^*}{\partial d_1} = V_{31}$ etc etc

6

$$\frac{\partial x^*}{\partial d_1} = \begin{bmatrix} V_{11} \\ V_{21} \\ V_{31} \end{bmatrix} = \frac{\partial}{\partial d_1} \begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix}$$

$$\frac{\partial x^*}{\partial d_2} = \begin{bmatrix} V_{12} \\ V_{22} \\ V_{32} \end{bmatrix} \qquad \frac{\partial x^*}{\partial d_3} = \begin{bmatrix} V_{13} \\ V_{23} \\ V_{33} \end{bmatrix}$$

$$\frac{\partial x^*}{\partial d} = V = (I - A)^{-1}$$

From $x^* = Vd$

we obtain $\frac{\partial x^*}{\partial d} = V$

Very neat!

L a display of all the comparative static derivatives

L answers the question: if planning targets $d_1, d_2 \ldots d_n$ are revised, how must we change output goals?

Jacobian Determinants

- We can test if functional (linear or otherwise) exists among a set of n functions

eg
$$y_1 = f^1(x_1, x_2, \ldots x_n)$$
$$y_2 = f^2(x_1, x_2, \ldots x_n)$$
$$\vdots$$
$$y_n = f^n(x_1, x_2, \ldots x_n)$$

The Jacobean determinant is:

$$|J| = \left| \frac{\partial(f^1, f^2, \ldots f^n)}{\partial(x_1, x_2, \ldots x_n)} \right|$$

$$|J| = \begin{vmatrix} f_1^1 & f_2^1 & \cdots & f_n^1 \\ f_1^2 & f_2^2 & \cdots & f_n^2 \\ \vdots & & & \\ f_1^n & f_2^n & \cdots & f_n^n \end{vmatrix}$$ where $f_j^i = \dfrac{\partial f^i}{\partial x_j}$

if $|J| = 0$, the equations $f^1, f^2, \ldots f^n$ are functionally dependent

1. Proof? We dont do it (complicated)

2. Why do we need this?
Sometimes we have equatio[ns] where we dont have specifi[c] / explicit functional form.
eg: $D(P, Y_0) - Q = 0$
$S(P) - Q = 0$
We dont know the form that $D(P, Y_0)$ takes, but still want to know $\frac{\partial D}{\partial P}$

for this, we need to know if functional dependence exists (more explanation later)

3. How does it tie into linear eQNS, eg $Ax = d$
Special case, $|A| = 0$ will be same as $|J| = 0$

Let's show this last pt:

Linear fns:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = d_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = d_2$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = d_n$$

- These are LD if $|A| = 0$
- What is $|J|$?

$|J|$ here $= \begin{vmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & & & \\ \frac{\partial y_n}{\partial x_1} & \frac{\partial y_n}{\partial x_2} & \cdots & \frac{\partial y_n}{\partial x_n} \end{vmatrix}$

Where $y_1 : a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n$

$\therefore |J| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$ This equals $|A|$

$\therefore$ if $|J| = |A| = 0$, functional (here linear) dependence exists

NB

Jacobian is for $n \times n$

If have $x_1, x_2, \ldots x_n, x_{n+1}, x_{n+2}$

Hold x_{n+1}, x_{n+2} as parameters

and proceed as usual.