Chapter 7

Theory of the Firm and Market Structure: Production

- The input output relationship/production function
- Production in the short run
- Production in the long run
- Returns to Scale

The above seems terribly dry.

What is production exactly?

Are the following **production workers?** Comedians, psychologists, singers, doctors, miners, gum boot manufacturers?

We're starting to focus on the **supply side**

What is the **decision process** which transforms **inputs into outputs**?

Questions I need to ask you:

What are **factors of production**? Examples?

What is a **production function**? Examples?

Does our production function change if **technology** changes?

What are the **features** of a production function?

Remind me about **diminishing marginal returns** for production please?

More questions

Short run and **long run** - what's that again?

What are **fixed** and **variable** inputs of production?

Examples of each?

Is the long run the same for every product?

Give me some products, and tell me their fixed and variable inputs - **in class.**

Where do goods and services come from?

Is Trevor Noah a production worker?

What is production?

http://1.bp.blogspot.com/_BNO37WEHKGo/TLYg2bC6dVI/AAAAAAAAAOk/U5xgQEb516M/s1600/Trevor.jpg

Production = any activity that creates present or future utility

The activities of doctors, lawyers, factories, rubbish collectors, apple growers etc etc = production

Production transforms inputs into outputs

http://i1.sndcdn.com/artworks-000015665533-fine18-crop.jpg?b988642

We now turn to focus on **Supply**

Chapters 1 - 6 talked about demand side factors

How does output vary depending on productive inputs, in the short and long run?

What can we do, **given the available technology and resources**?

Inputs = factors of production.

Factors of production =

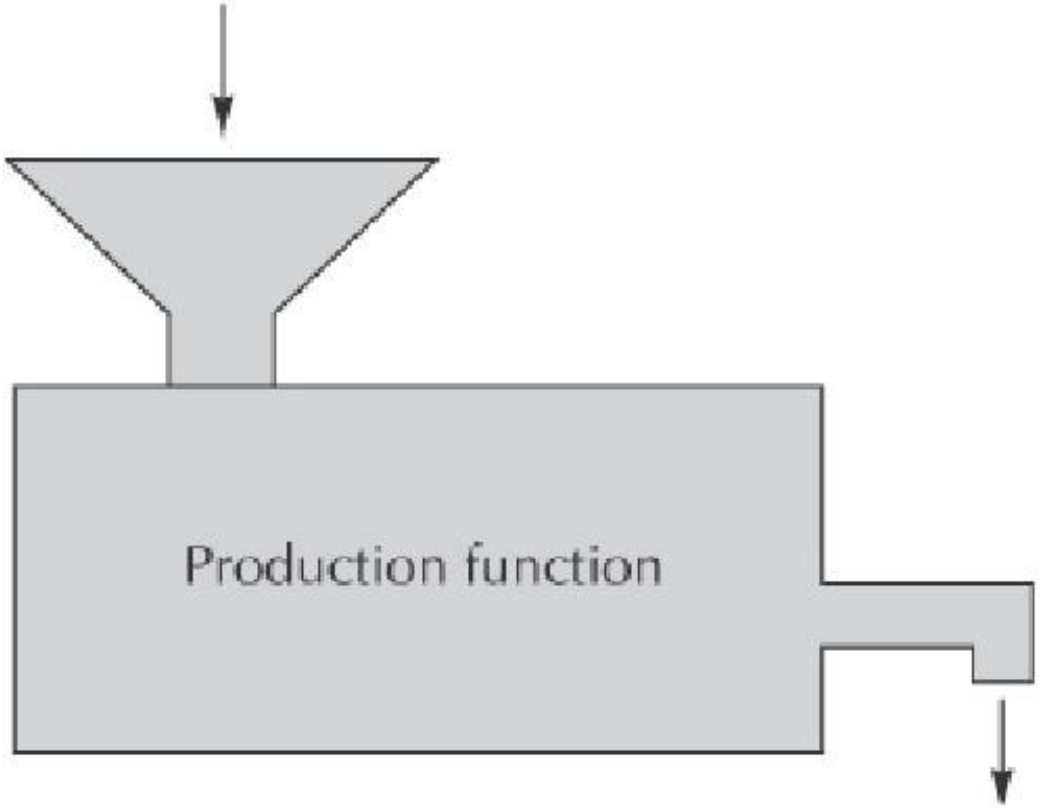
Land
Labour
Capital
Entrepreneurship
technology/knowledge/organisation/energy

Production Function = Relationship by which our inputs turn magically into outputs

Better technology = better box - see diagram

Figure 7.2: The Production Function

Inputs
Land, labour, capital, and so forth



Outputs
(cars, polio vaccine,
home-cooked meals,
TV broadcasts, and so forth)

Production function = recipe

Double the ingredients, do you get double the cake?

Can we substitute ingredients? Must we keep the same proportions?

http://www.flickr.com/photos/thagoodiez/5486836414/sizes/l/in/photostream/

$$Q = f(K, L)$$

Q = output
K = **capital**, L = **labour**

How much Q do you get, when you use certain amounts of K and L. E.g.

Q = 2KL
L = a **student**, K = a CD **printer**,
Q = number of pirated CDs

If L = 3, K = 2, Q = 12
We ignore **intermediate products** in Chapter 7

Q = 2KL		Number of Students = L				
Number of CD Presses = K		**1**	**2**	**3**	**4**	**5**
	1	2	4	6	8	10
	2	4	8	12	16	20
	3	6	12	18	24	30
	4	8	16	24	32	40
	5	10	20	30	40	50

Fixed and **Variable** Inputs

Variable inputs - can be changed or increased
- its quantity can be changed in the **short run**

Fixed inputs cannot

Output = operation

Inputs = anaesthetic, vascular surgeon

Which is which?

http://www.flickr.com/photos/ale_era/3681831618/sizes/z/in/photostream/

Long run = shortest period of time required to alter the amounts of every input

For stroke operations, the long run is 15 years - the amount of time required to train vascular surgeons

Short run = the period during which **one or more inputs can't be varied**

Is the long run the same for every product?

Apples = **output**
Apple pickers = **variable input**
Land = **fixed input**

CDs = output
Students = variable input
CD press = fixed input

Slap chips = output
Chip fryer = variable input
Deep fryer = fixed input

http://www.growersguidetocannabis.com/blog/wp-content/uploads/2011/11/pic00151.jpg

Lecture 7 Concepts

- SR, LR - fixed, variable
- marginal product - w.r.t. K, L
- if we're changing K, are we in the SR?
- Production fns in SR = fn (?)
- Exact defn of diminishing returns
- where do we see marginal returns/output decline on the graph?
- Who was Malthus and why was he worried?
- How do prod fns change when tech changes?
- TP,MP, AP
- How do we decide how much to produce?

Production in the Short Run

$$Q = F(K,L) = 2KL$$

Which is fixed in the short run? Variable?

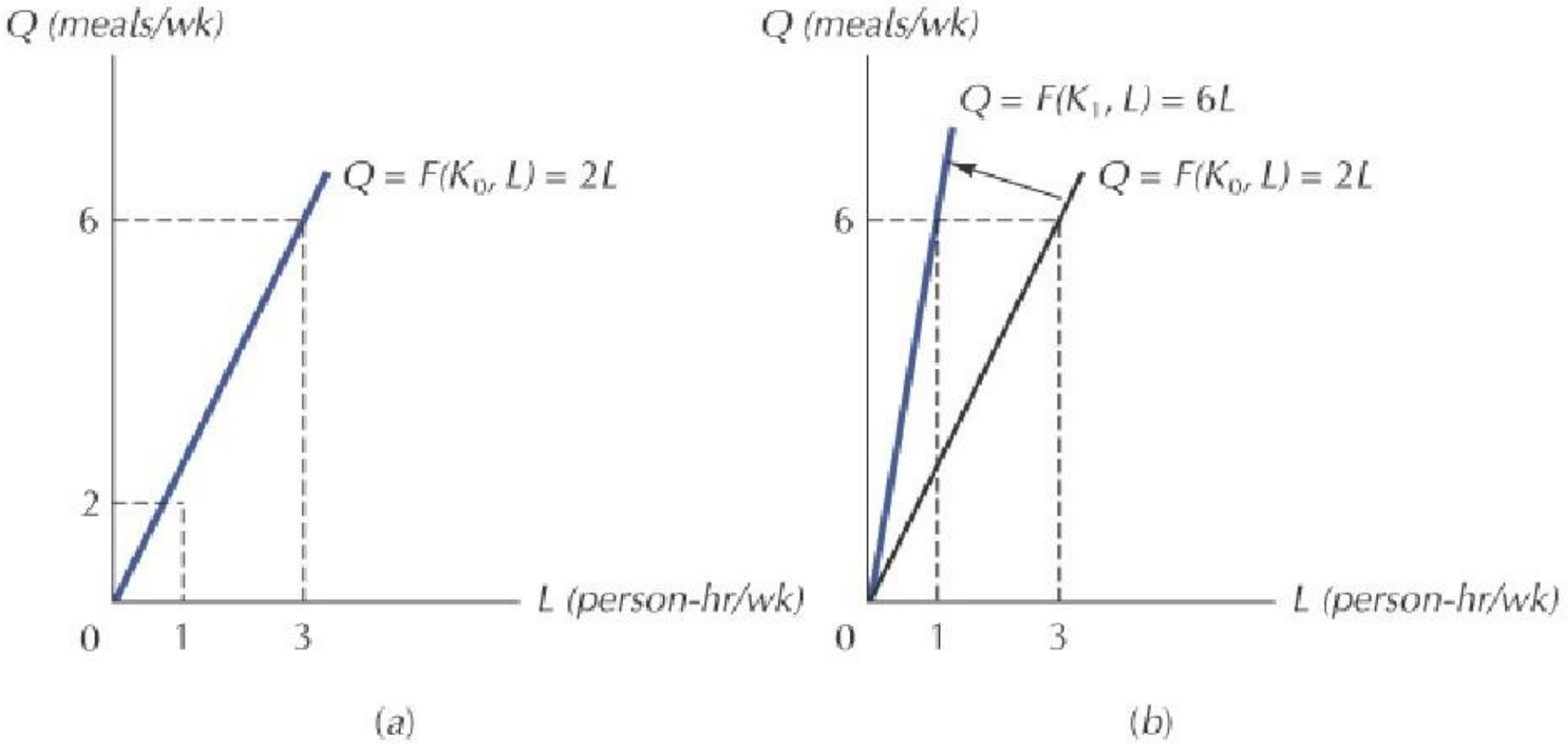
$$K = K_0 = 1$$

Therefore $Q = F(K,L) = 2K_0L = 2L$

$$\Delta Q/\Delta L = 2K_0$$

What if I increase the amount of K **?**

Figure 7.3: A Specific Short-Run Production Function



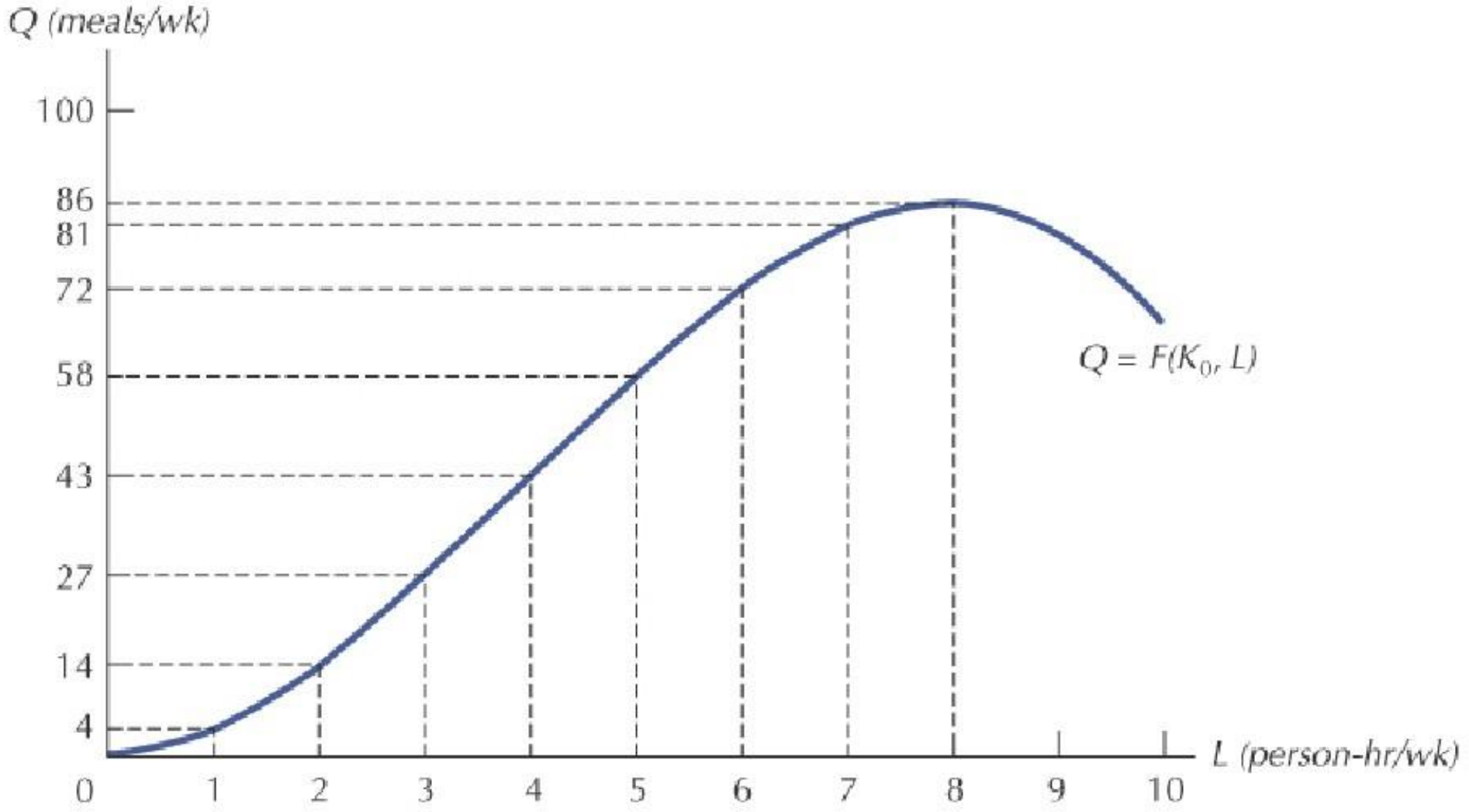
Are short run production functions always straight lines?

Properties of Production Functions:

- they pass through the origin. Why?
- initially increasing inputs increases output at an increasing rate
- beyond some point, extra inputs increase output at a decreasing rate, or even decrease output
- why does output increase at an increasing rate

?

Figure 7.4: Another Short-Run Production Function



As you add more and more students to the dodgy CD piracy flat, eventually the number of pirated CDs being produced will start to fall.

:(

What do we call this? Why does it happen?

?

Law of diminishing returns

If other inputs are **fixed**, the increase in output from an **increase in the variable inputs** must eventually decline

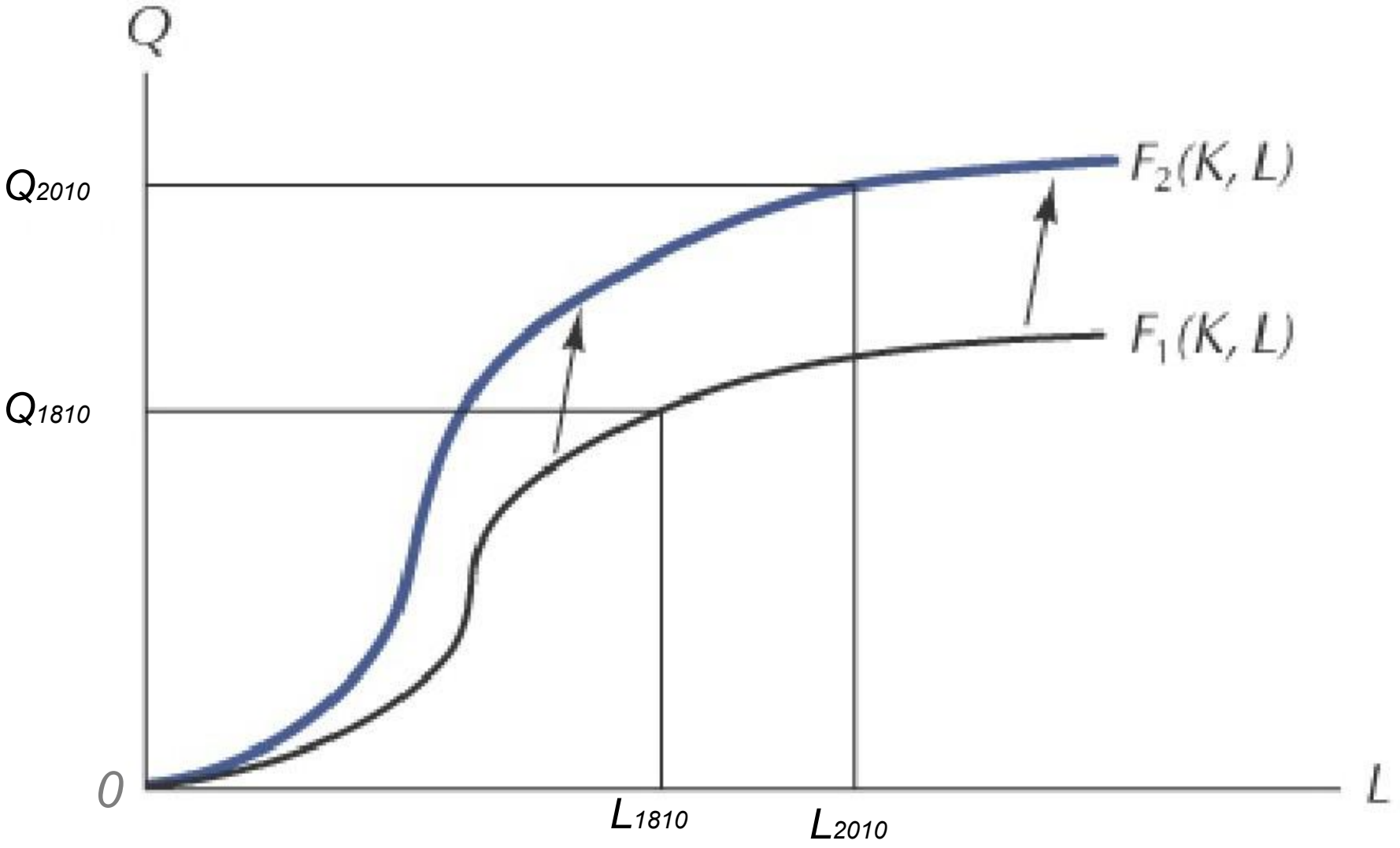
?

How common is this?

Is it a short or long run phenomenon?

According to **Malthus**, does this mean we are in trouble? Were his predictions wrong?

Figure 7.5: The Effect of Technological Progress in Food Production



How do we represent technological improvements in production on the graph?

CLARIFICATION

Q: Where do **marginal returns** start to diminish?

A: After L = 4

Q: Where does **output** start to diminish?

A: After L = 8

diminishing **returns**
=
diminishing **marginal product**

see p 231

Remember:

Diminishing returns = short run concept

Decreasing returns = long run concept

For functions with decreasing, constant or increasing returns to scale, in the short run they will most likely display diminishing returns.

Total, **Marginal** & **Average** Product

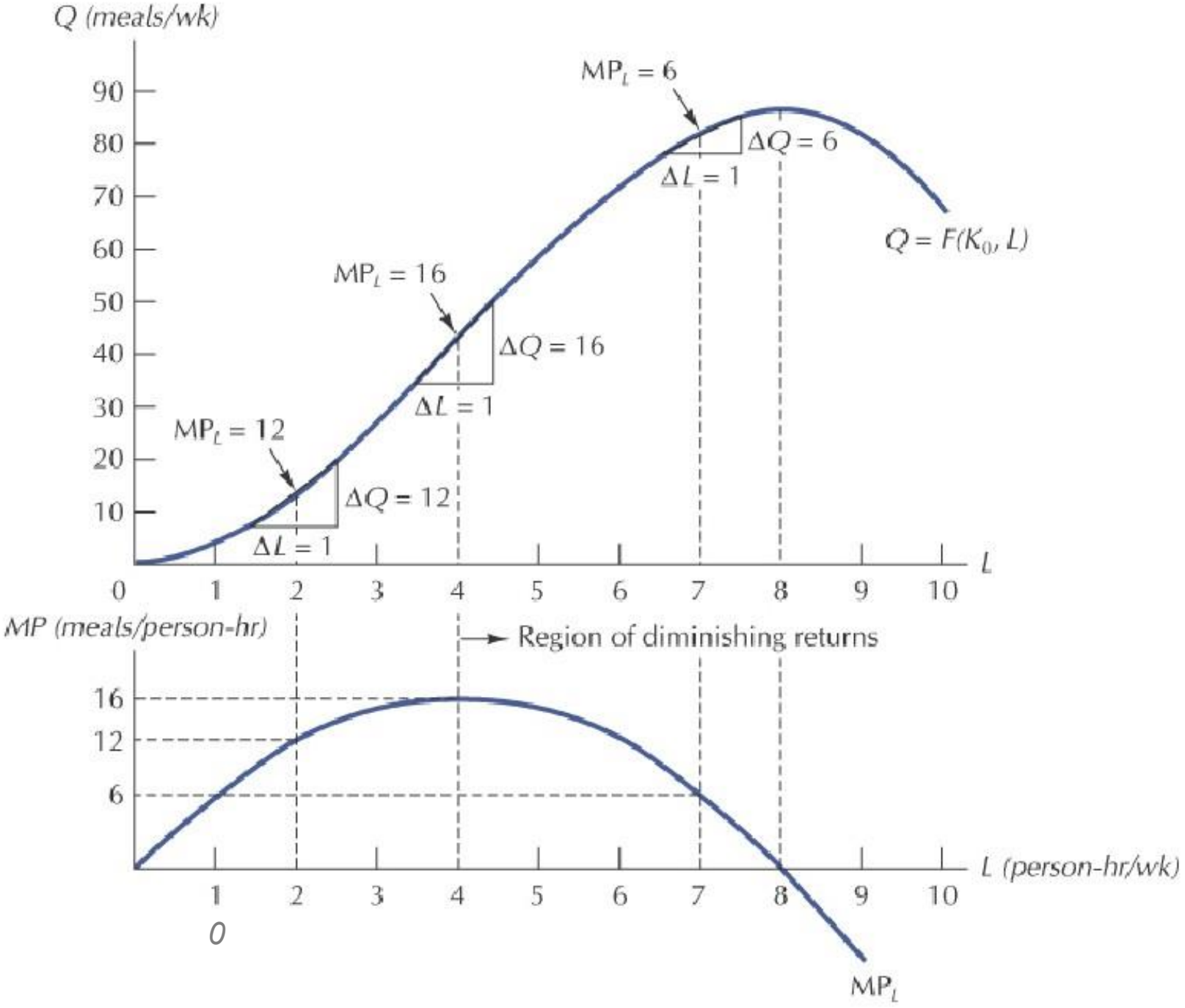
Total product curve - relates the total amount of output to the quantity of the variable input

Marginal product - the change in total product that occurs when we increase the variable input by one unit (while holding other inputs fixed)

Did I need to say that last bit? Is MP important?

$MP_L = \Delta Q/\Delta L = dQ/dL$

How do we represent MP geometrically?


Q (meals/wk)
90
80
70
60
50
40
30
20
10
MP_L = 6
ΔQ = 6
ΔL = 1
Q = F(K_0, L)
MP_L = 16
ΔQ = 16
ΔL = 1
MP_L = 12
ΔQ = 12
ΔL = 1
0 1 2 3 4 5 6 7 8 9 10 L
MP (meals/person-hr)
Region of diminishing returns
16
12
6
1 2 3 4 5 6 7 8 9 10 L (person-hr/wk)
0
MP_L

What does dodgy student Joe do?

What is MP_L when L = 2? and L = 4? L = 7?

Is the TP curve **convex** or **concave**?

Where is **maximum** MP_L? & maximum TP?

Should we increase L beyond 8?

Is all this graphical/mathematical stuff important?

YES!

Average Product = total product per unit of variable input

$AP_L = Q/L$

E.g. AP_L = 20 CDs per student

With labour, we call this **labour productivity**

Does AP change depending on where we are on the TP curve? Where is AP at a maximum?
How do we represent AP geometrically?
Can AP be the same at different places on the TP curve?

Summary Slide: Formulae

TP: $Q = f(L)$

MP: dQ/dL

AP: Q/L

Summary Slide: Graphically

TP: the graph of Q = f(L)

MP: the slope of the graph (i.e. the slope of a tangent to the graph)

AP: the slope of the ray/chord from the origin to a point on the graph

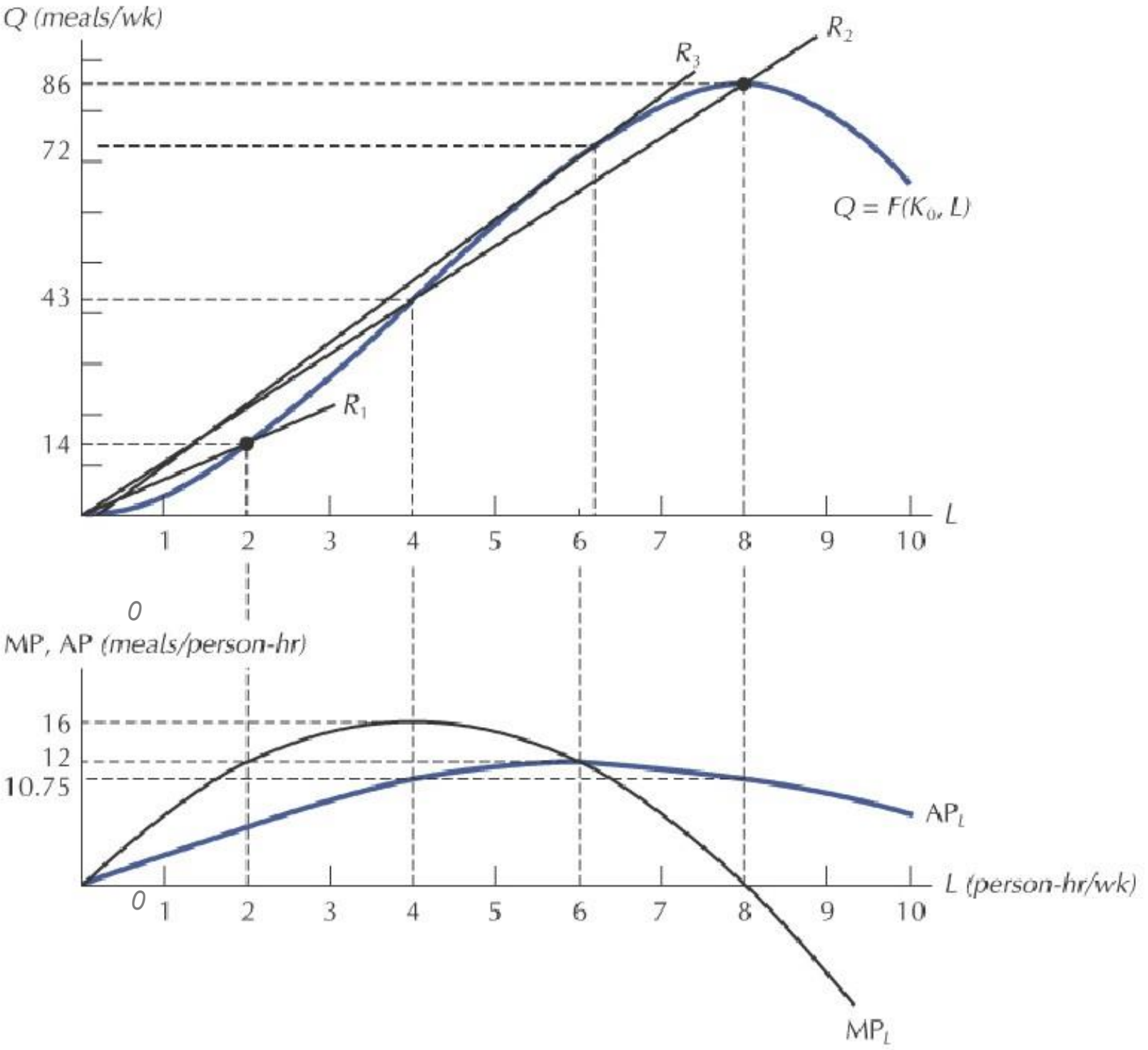
You are the CEO of Burgercom Megacorp

Do you hire another person to flip burgers?

decide based on changes - you **compare benefits & costs of an additional unit of input**.

This is MP.

With fig 7.6, should you stop hiring at some point?


Q (meals/wk)
86
72
43
14
R3
R2
R1
Q = F(K0, L)
L
1
2
3
4
5
6
7
8
9
10
0
MP, AP (meals/person-hr)
16
12
10.75
APL
L (person-hr/wk)
0
MPL

How are MP, AP and TP related?

MP = slope of a tangent to the TP curve
AP = slope of chord from origin to a pt on TP curve

Where is AP > MP? And AP < MP?

Where does AP = MP?

If MP lies above AP, is AP rising or falling?

?

If L = 0, what is MP? And AP?

If the contribution to output of an additional unit of the variable input (MP) exceeds the average contribution of the variable inputs (AP) used thus far, the average contribution must rise (AP)

Put differently:

If I add clever clogs Norman to the average intelligence math club, the average intelligence level rises.

How to allocate scarce resources

Organic Hey Shoo vs Big Agriculture

How should we grow apples?

Or why do we care about the difference between AP and MP?

AP, MP, TP: organic & conventional

	Organic			Conventional		
Number of Apple Pickers	**AP**	**TP**	**MP**	**AP**	**TP**	**MP**
0		0			0	
1		100			130	
2		200			240	
3		300			330	
4		400			400	

AP, MP, TP: organic & conventional

	Organic			Conventional		
Number of Apple Pickers	**AP**	**TP**	**MP**	**AP**	**TP**	**MP**
0	0	0		0	0	
1	100	100	100	130	130	130
2	100	200	100	120	240	110
3	100	300	100	110	330	90
4	100	400	100	100	400	70

The life of an apple farmer...

Should all 4 apple pickers work in organic?

If I have 2 apple pickers growing organic, and 2 conventional, should I make any changes?

What is the difference between organic and conventional growth?

What is the optimal mix?

How do I figure this out?

Rules:

To allocate an input efficiently, allocate the next unit of the input to the production activity where its **MP is the highest**

For a resource that is **perfectly divisible**, allocate the resource so its MP is the same in every activity (e.g. hours spent by 1 picker)

If the organic or conventional land amounts are limited in size, the third picker will crowd out some of the production of the other 2 pickers.

Do you need to know about tennis?

Production Function for kilos of apples:

$$Q = 30 + 70L - 90L_2$$

L = weeks spent growing apples

1. where is Q maximised?
2. What is the maximum value of Q?
3. What is an expression for MP_L?
3. What is the MP_L at the point in 1.?
4. What is an expression for AP?
5. What is AP at L = 0.2?

Production in the Long Run

Short Run Prodn - at least one input is fixed
Long run - all inputs are variable

We need more than 2 dimensions to graph this

$$Q = F(K,L) = 2KL$$

For a particular output level Q_0

$$2KL = Q_0 \text{ therefore } K = Q_0/2L$$

E.g. $Q_0 = 16$, $K = 8/L$

The more general form:

$$K = C/L$$

Where C is some positive constant

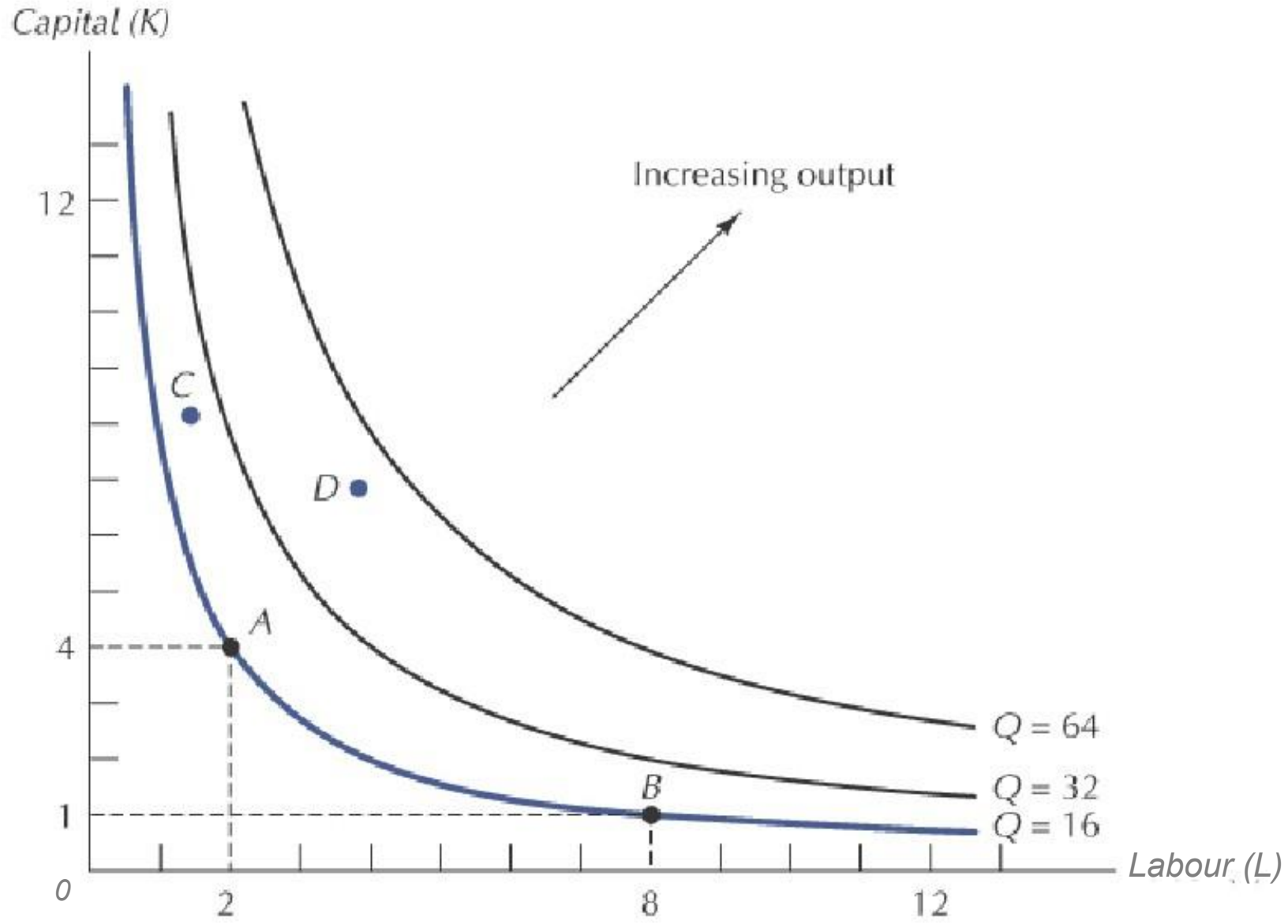
This is a **hyperbola**, defined where K, L, C > 0

We graph these as isoquants

Isoquants - also called **equal product curves:**

all combinations of variable inputs that yield a given level of output

Figure 7.11: Part of an Isoquant Map for the Production Function $Q = 2KL$



Which direction does overall production increase in?

Indifference curves showed **relative** rankings

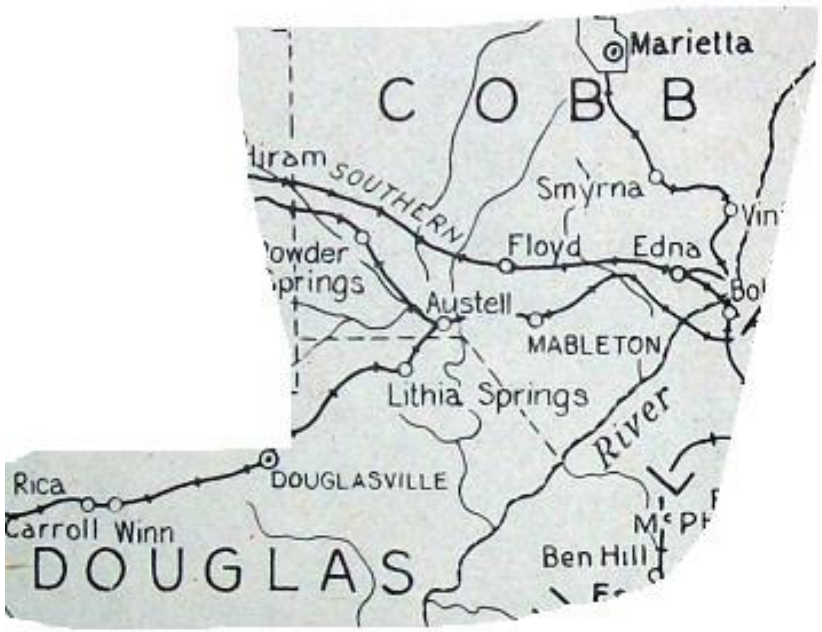
Isoquants can actually show **absolute** rankings

Can we **relabel** isoquants? And indifference curves?

What is on the axes for both types of curves?

?

Prodn Fns



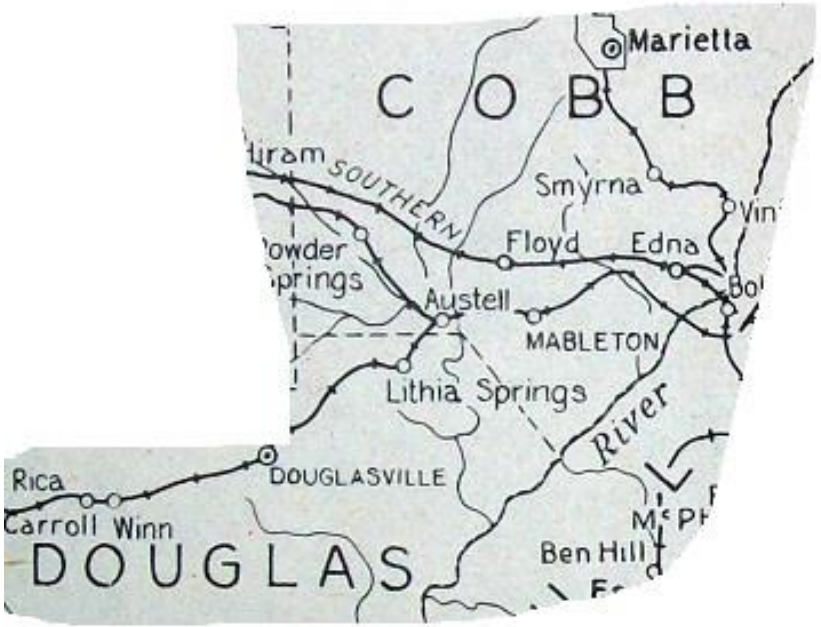
Cobb Douglas Prodn Fn:

$Q = mK^{\alpha}L^{\beta}$

How to solve for a function K = fn(L)?

And for $Q=K^{1/2}L^{1/2}$? What about MP_K? MP_L?

Prodn Fns



Cobb Douglas Prodn Fn:

$$Q = mK^{\alpha}L^{\beta}$$

set $Q = Q_0$, and solve for K in terms of L

$$Q = mK^{\alpha}L^{\beta} = Q_0$$

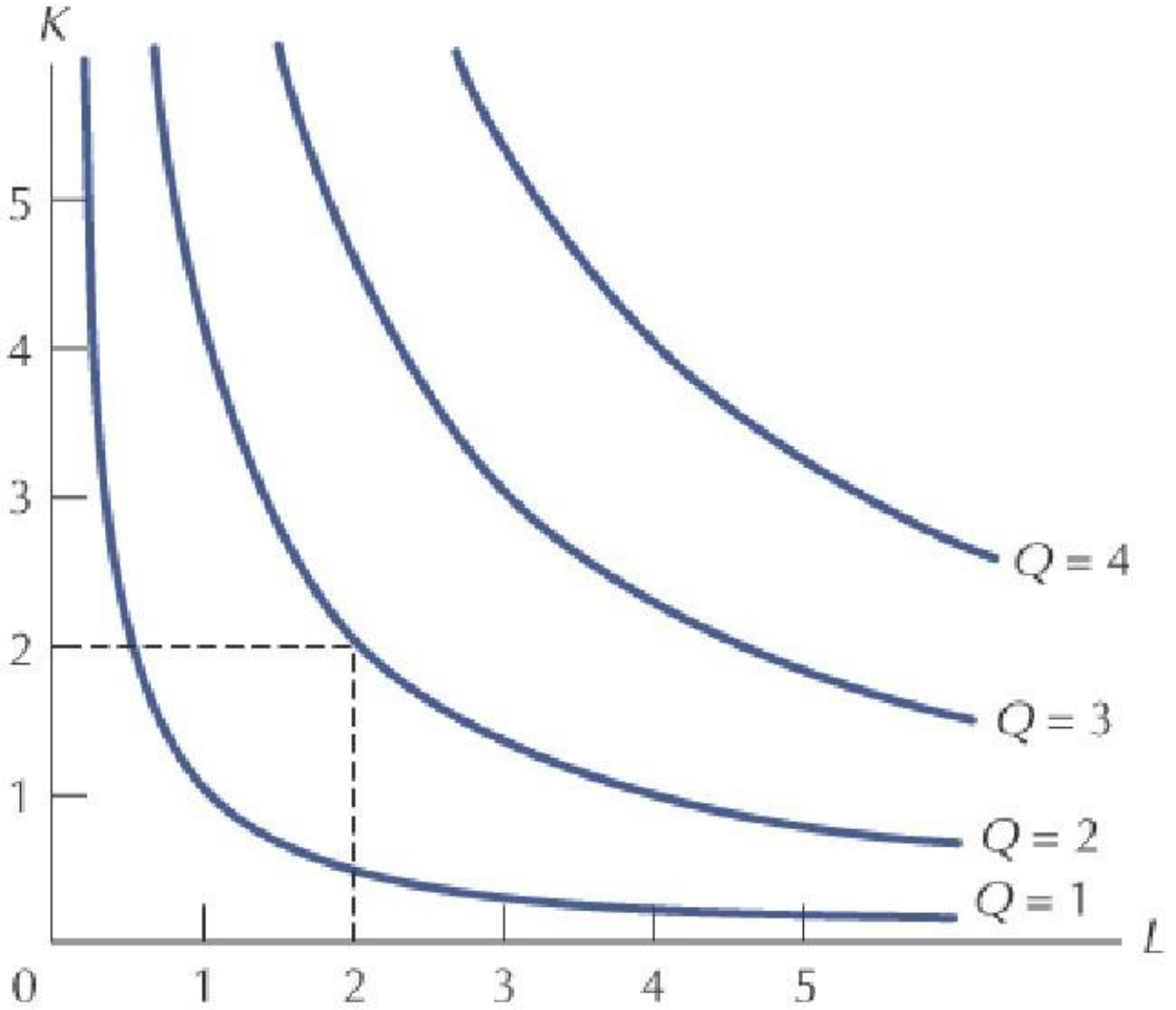
$$K^{\alpha} = Q_0 /(mL^{\beta})$$

$$K = Q_0^{1/\alpha} /(mL^{\beta})^{1/\alpha}$$

different to the textbook - is that okay?

For $Q = K^{1/2}L^{1/2}$, we get $K = Q_0^2/L$

Figure 7.12: Isoquant Map for the Cobb-Douglas Production Function $Q = K^{1/2}L^{1/2}$



Thank goodness for commerce students...

$Q = mK^{\alpha}L^{\beta}$

What is MP_K? MP_L?

$\mathbf{MP_K = \partial Q/\partial K} = \alpha mK^{\alpha-1}L^{\beta}$

$\mathbf{MP_L = \partial Q/\partial L} = \beta mK^{\alpha}L^{\beta-1}$

You **MUST** know how to calculate these expressions (don't memorise formulae)

http://www.commerce.uct.ac.za/Commerce/News/2008/20081102_AmericanVote.jpg

Lecture 8 concepts

MRTS - defn, relnship to MPs
Leontief production function
Returns to Scale (Mathematically, Graphically)
Decreasing returns?

Wrapping up:
Tennis, rugby? Not important
Perfect complements and substitutes - realistic?

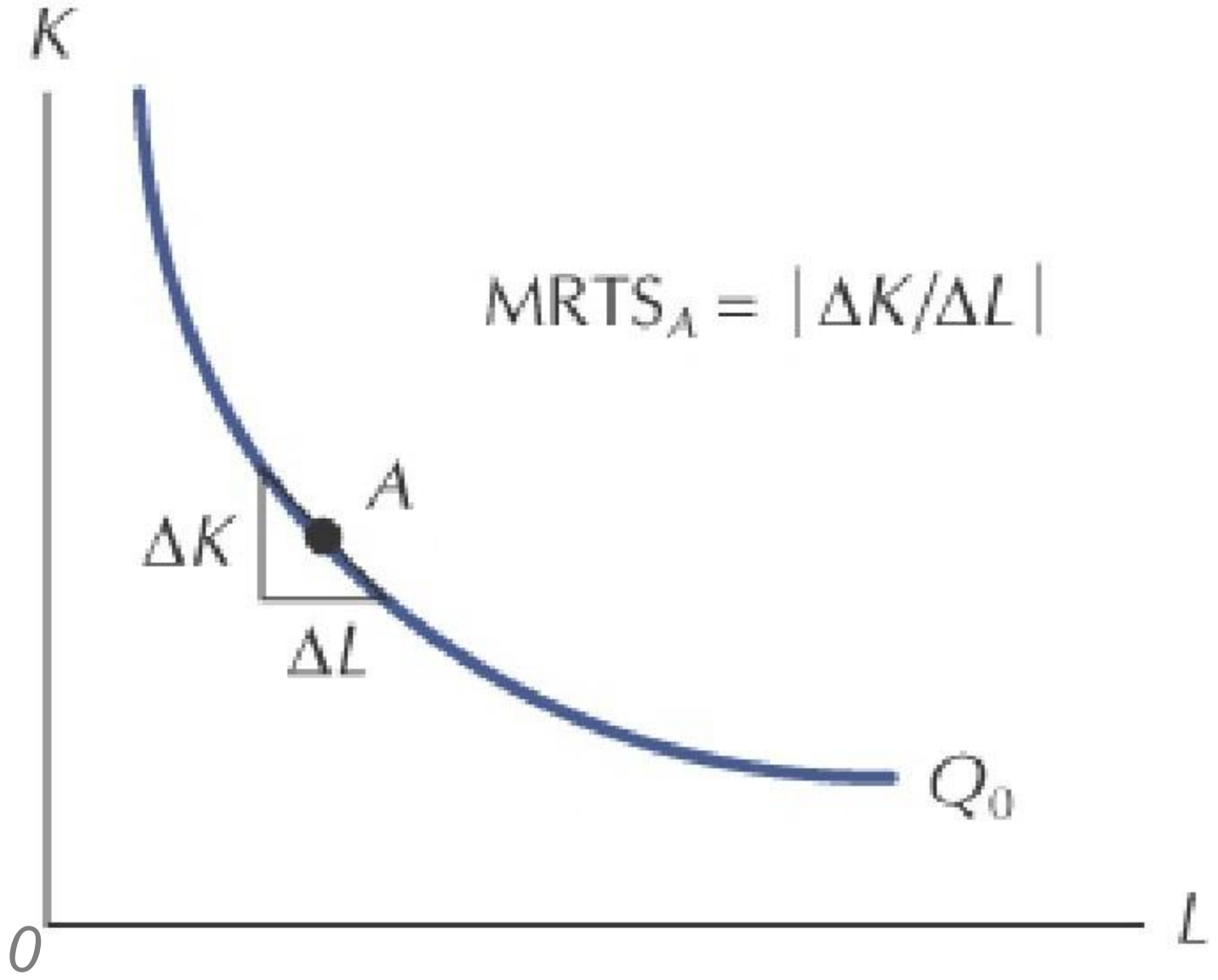
We wanted to ask how we can change K or L in relation to each other, while keeping output constant.

i.e. How can we **substitute labour for capital** (or v.v.) while staying on the **same isoquant**?

We know the rate of substitution changes depending on where we are on the isoquant (remember what happens as L → infinity, L → 0)

This is called the **marginal rate of technical substitution - MRTS**

if I reduce L by ΔL, I must increase K by ΔK, to keep output constant



MRTS - rate at which one input can be exchanged for another, **without altering output**

If MRTS = 5,
i.e. MRTS = $\Delta K/\Delta L = 5/1$,
I can keep my number of CDs constant, while swapping 5 CD presses for 1 student

?

Why is the **absolute value** used?

MRTS = **slope of the isoquant**

i.e. the fewer CD presses we have, the more students we must add to keep output constant

At point A, if I reduce K slightly - by ΔK, then in order to keep output the same, I must increase L slightly - by ΔL

To stay on the same isoquant (i.e. **to keep output constant**), the change in output from reducing K, must equal to the change in output from increasing L

N.B. $MP_K.\Delta K = MP_L.\Delta L$

Therefore MRTS = $MP_L/MP_K = \Delta K/\Delta L$

$MRTS = MP_L/MP_K = \Delta K/\Delta L$

MRTS = slope of the isoquant

As we use more and more labour

$MP_L \rightarrow 0$

Therefore $MRTS \rightarrow 0$

(i.e. on **flat portions** of the isoquant, **MRTS = 0**)

As we use more and more K (i.e. $L \rightarrow 0$)

$MP_K \rightarrow 0$

Therefore $MRTS \rightarrow$ infinity

(i.e. on vertical portions of the isoquant, $MRTS \rightarrow \infty$)

Substitutes in Production

You can create an **internet** connection using either:

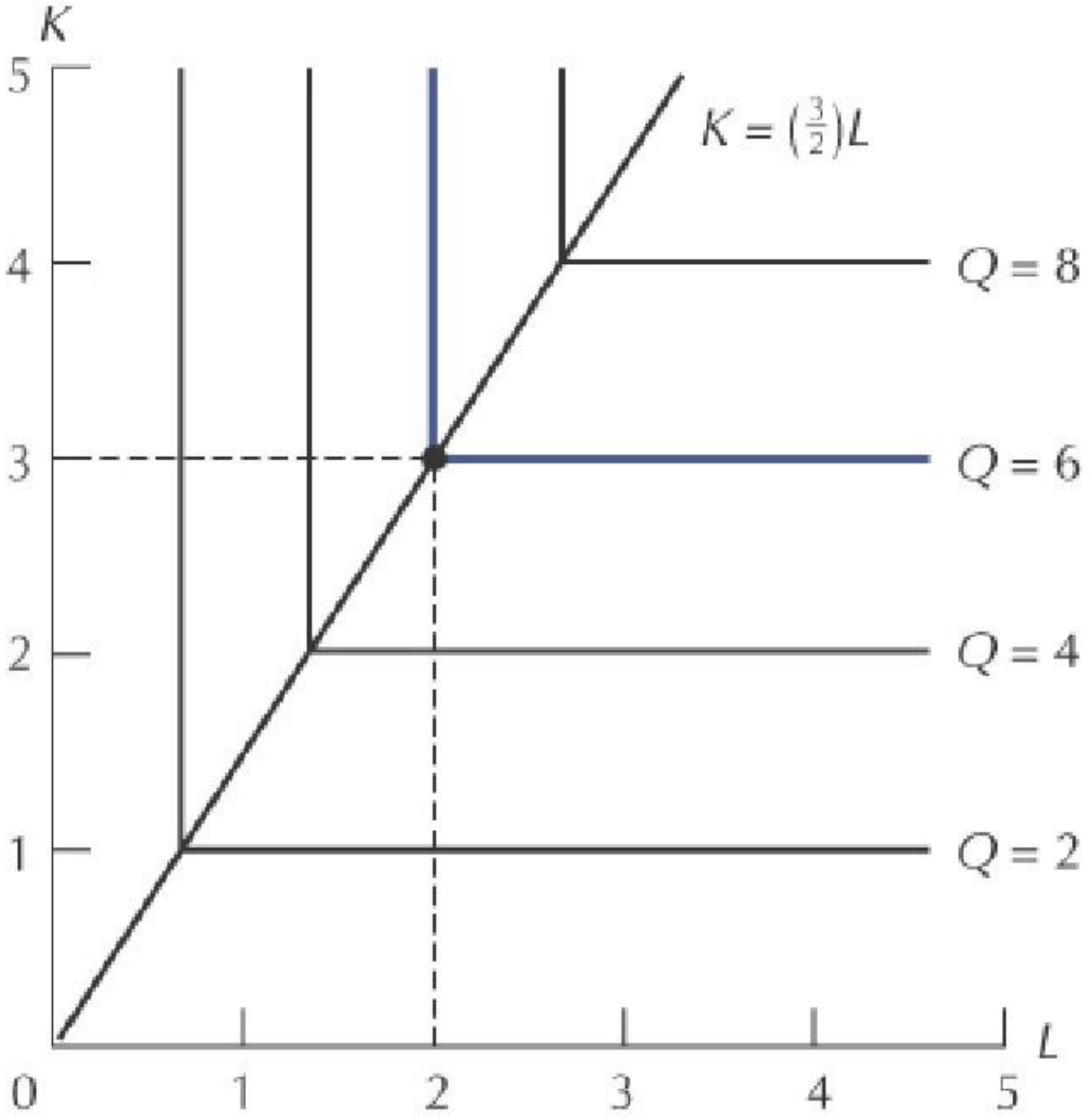
Satellite OR Fibre Optic Cable

Compliments in Production

To make an **internet** connection, you need:

Phone Lines + Computers

More computers will not help you if you do not have any phone lines!

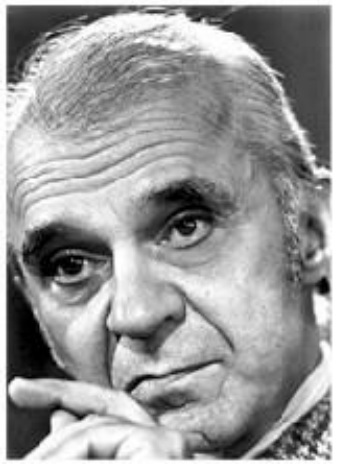


Leontief Production Function

Q = min (2*K*,3*L*)

Perfect **Complements**

Fixed Proportions



Why is the **Leontief** production function called the **fixed proportions fn**?

$$MRTS_{K/L} = MP_L/MP_K = \Delta K/\Delta L$$

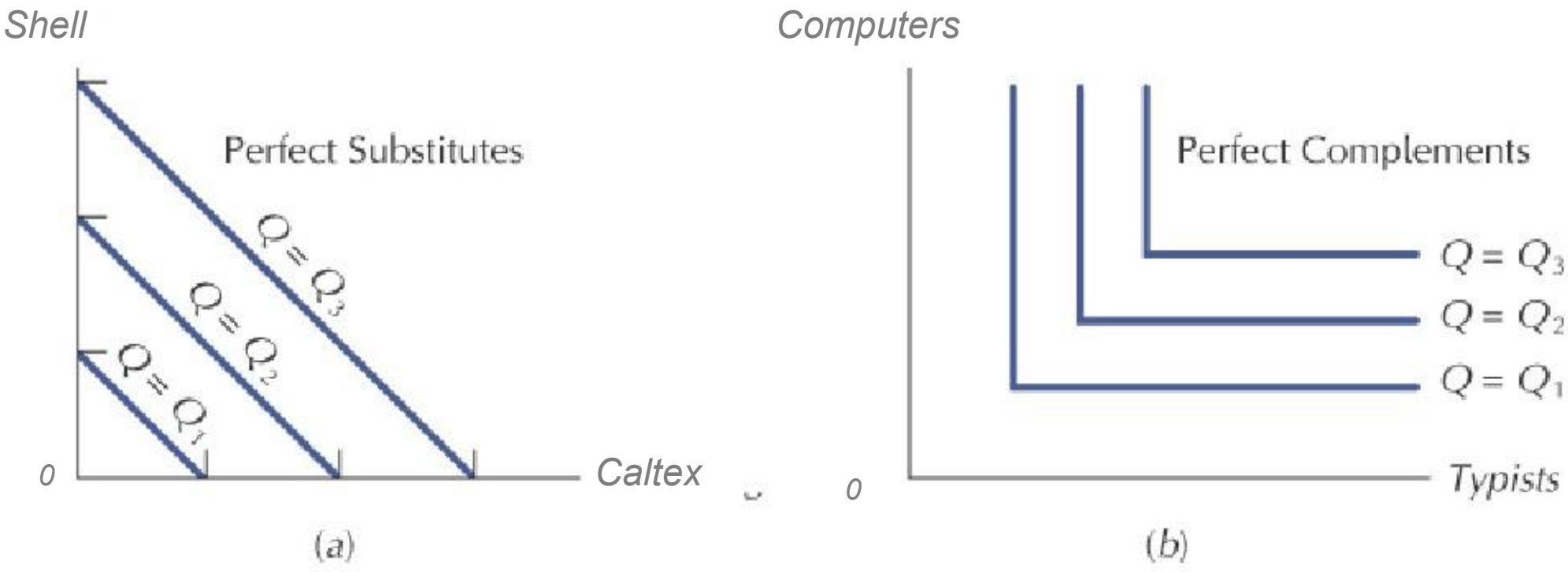
What is the **MRTS** on the vertical arm? And the horizontal? And at the cusp?

Answers are infinite, zero, and undefined.

Why?

?

Figure 7.15: Isoquant Maps for Perfect Substitutes and Perfect Complements



Returns to Scale

As I scale up production (i.e. use more K and L), what happens to output?

If I double inputs, do I get double output? Or more than double?

If more than double, I should scale up production, - this is **increasing returns to scale**

Why do we get increasing RTS? **Specialisation.**

Returns to Scale

RTS = **long run concept**

Increasing RTS: increase inputs by 1%, output will increase by more than 1%

E.g. should an airline be run with low numbers of workers and planes, or large numbers? Where will they be **more efficient**?

Increasing RTS - **will see fewer larger firms. Why?**

Returns to Scale
& Economies of Scale.

The same? Different?

Returns to scale - inputs and production in LR
Economies of Scale - costs (average cost declines as we produce more).

In general,
increasing returns to scale = economies of scale

(under perfect competition in factor markets).

Mathematical Defn of Returns to Scale

We have $Q = f(K,L)$

How much Q do we get if we multiply K and L by c

What is $f(cK, cL)$?

Increasing RTS: $f(cK, cL) > cf(K,L)$
Constant RTS: $f(cK, cL) = cf(K,L)$
Decreasing RTS: $f(cK, cL) < cf(K,L)$

Does this function display increasing returns?

E.g. $Q = F(K,L) = 2KL$

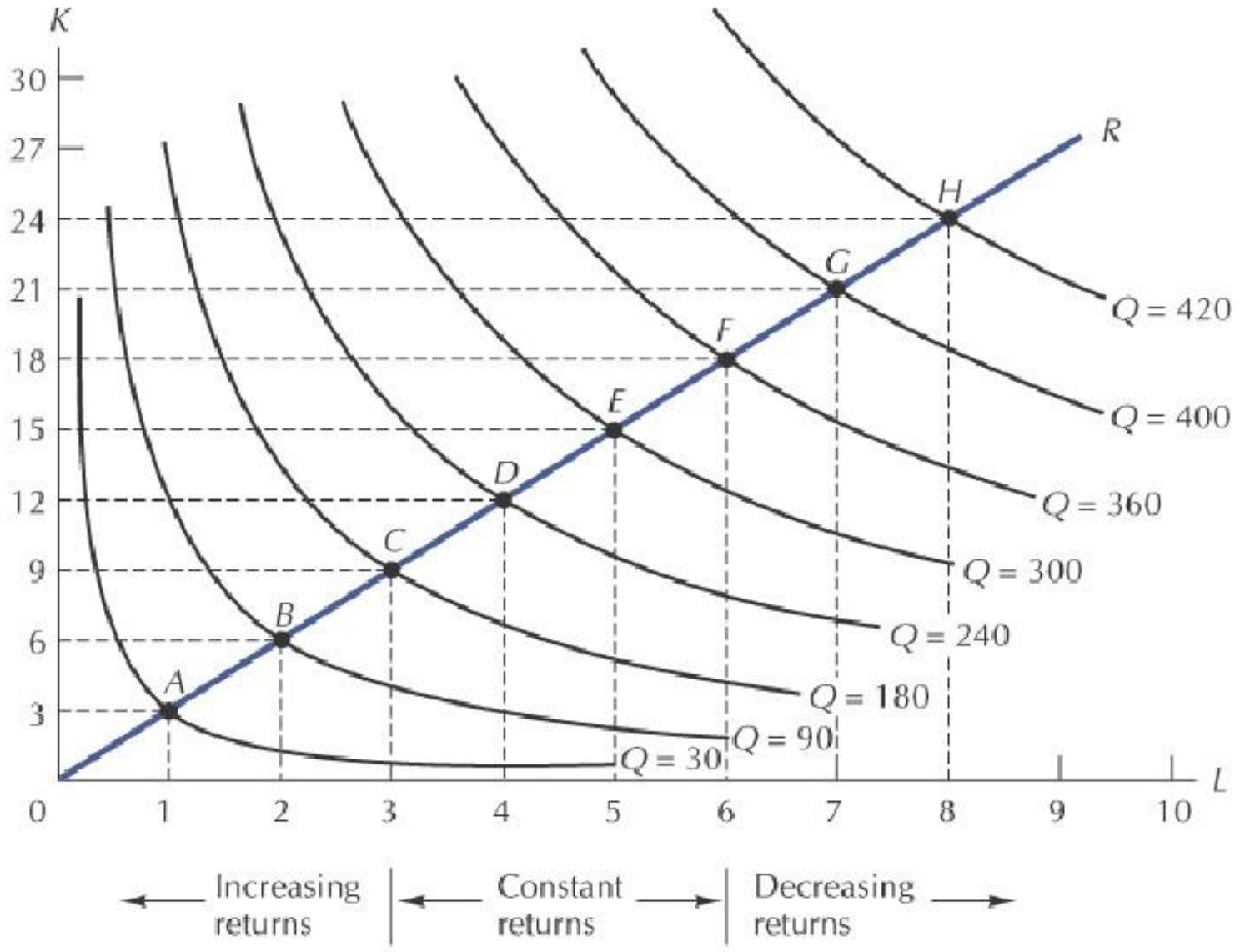
Multiply K and L each by c

$F(cL, cK) = 2(cK)(cL) = c_2 2KL > cF(K,L)$

Thus increasing returns to scale

Try with c = 2, c = 3 etc.

Figure 7.16: Returns to Scale Shown on the Isoquant Map



Decreasing Returns to Scale - a fiction?

Is it possible that any production function could ever display decreasing RTS?

We could just replicate the factory - double the K and L - and then get double output

Why in practice can we not do this?